A *Very Basic* Introduction To Time/Frequency Domains

Particle

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Abstract

A very brief introduction to waves, terminology, time/frequency domains, with a bit of mention of various transforms.

1 Introduction

In the context of communications, a signal is basically some information somehow encoded as a wave. Everything travels as a wave, so let's look at some of the simple ideas of that. Figure 1 presents a basic Sin wave.

The vertical axis is the volume, or air-pressure, or strength, or energy, etc., depending on what this wave is. For example, if this is an ocean wave, then the vertical axis may be the wave height. If this is a sound wave in the air, then it is the air-pressure. If it is the sound wave in a wire, then it's the voltage.

The horizontal axis (here depicted in terms of π) often represents the *time* or *distance* (or *space*) that the wave traverses in.

Waves generally have a period, which can be viewed as the time/distance of when the wave repeats itself—starts the next iteration. In Figure 1 the period is 2π.

A closely related property of the period is the *frequency*. Frequency is the speed at which the wave cycles its periods, and is always expressed in proper speed units, as in 'cycles per second' (commonly known as Hz), and 'kilo-cycles per second' (commonly known as kHz)[1].

[1] Naturally 'kilo' stands for 1000, so if something is 8kHz, then it's 8000Hz.

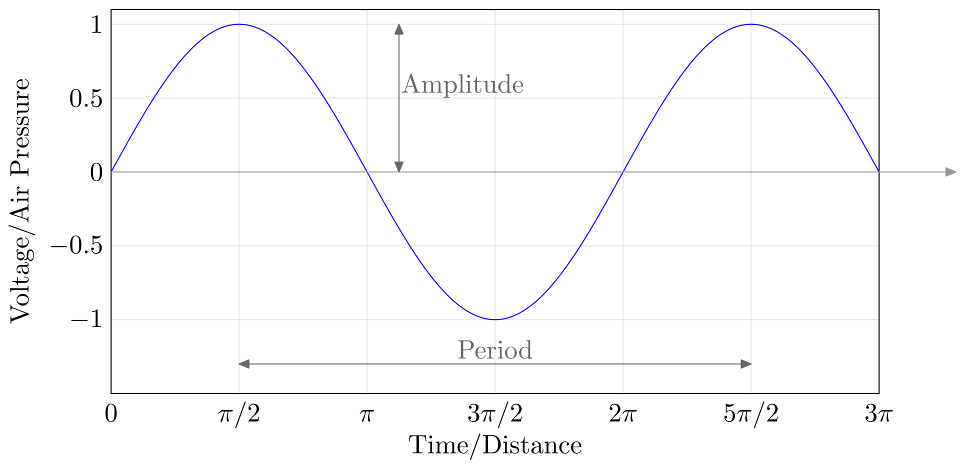

Figure 1: Analog Sin Wave.

2 Time Domain

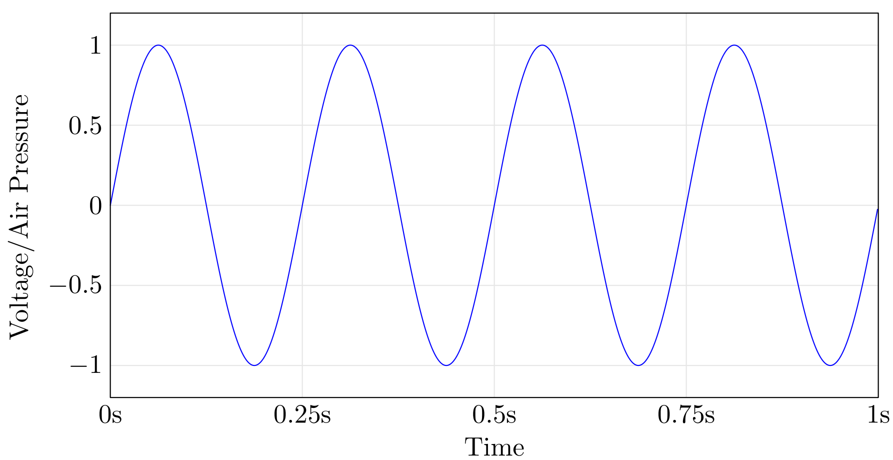

Figure 2: 4Hz Sin Wave; $\sin(2\pi 4t)$

Figure 2 illustrates a *Sin* wave at 4Hz. Notice that the horizontal axis is now *time*, and is expressed in *seconds*. You can count the number of times the wave cycles, and you'll see that it's 4, ie: 4Hz.

Given the frequency, you can easily find the wave period. For example, if the wave cycles itself 4 times per second, then the period is $\frac{1}{4}$ seconds, or simply $1/Frequency$. Similarly, knowing the period, you can find the frequency.

Another closely related value is the *wavelength*, which is basically the distance occupied by the wave period. Given the speed of propagation (light in a vacuum, or sound in the air, etc.), we can calculate that too. For example, light/electricity travel around 3×10^8 meters per second. Let's imagine that Figure 2 represents electricity in a wire, then the wavelength is just $3 \times 10^8 \div 4$ or just 7.5×10^7 meters, or simply $velocity \div frequency$. Obviously you can find velocity if you know the wavelength and frequency.

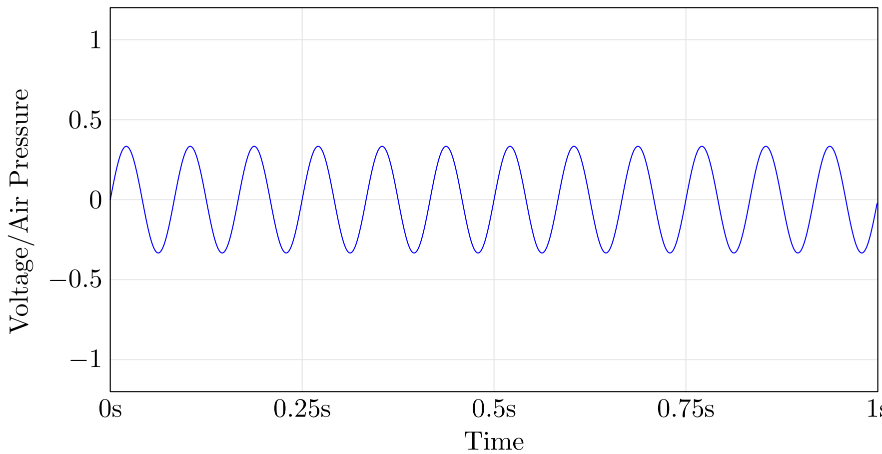

Figure 3: 12Hz Sin Wave; $\frac{1}{3\sin(2\pi 12t)}$

Continuing on with the discussion, Figure 3 shows a wave of 12Hz, and with $\frac{1}{3}$ amplitude. Notice how the period got smaller as the frequency increased.

3 Frequency Domain

Figure 4 illustrates the idea of taking two waves and adding them together. In this case, we've added waves from Figures 2 and 3.

There are primarily two ways of viewing any type of a wave; in the *time* domain, or in the *frequency* domain. The frequency domain of Figure 4 might

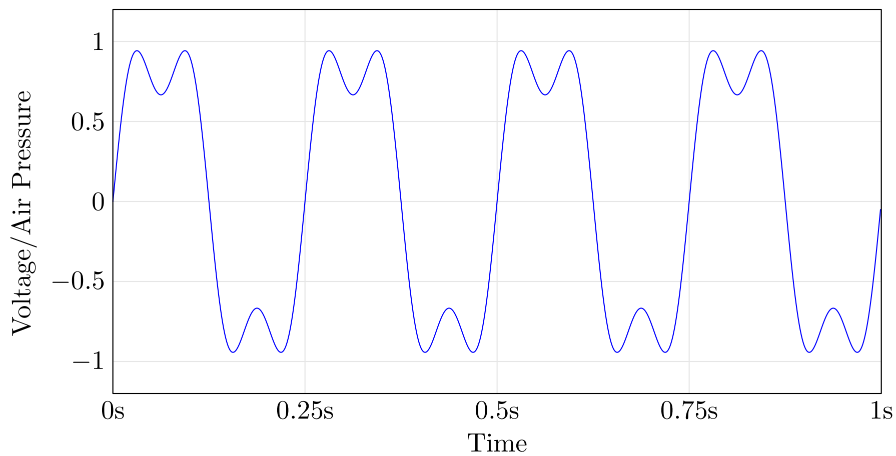

Figure 4: 4Hz + 12Hz Sin Wave.

look something like Figure 5.

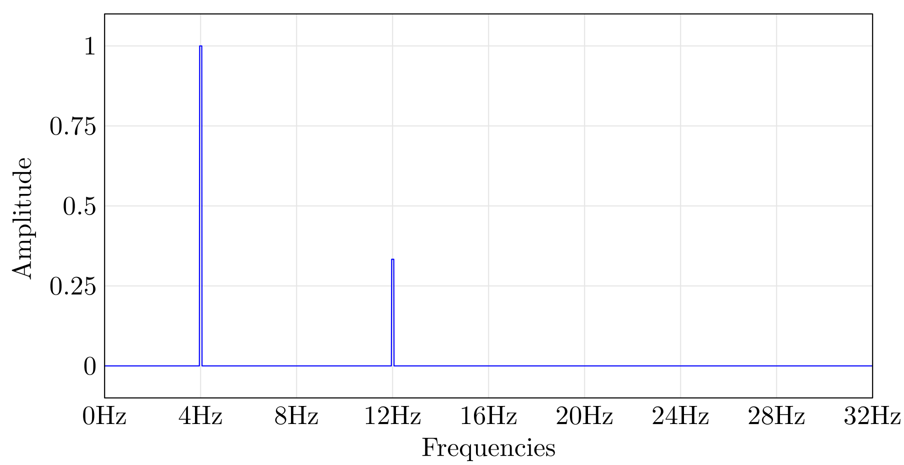

Figure 5: Frequency Domain of 4Hz + 12Hz Sin Waves.

4 Applications

It turns out that viewing waves in frequency domain is usually a lot more useful than viewing them in the time domain.

5 Discrete Fourier Transform

$$H_n = \sum_{k=0}^{N-1} h_k e^{2\pi ikn/N}$$

6 Inverse Discrete Fourier Transform

$$h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi ikn/N}$$

7 2D Discrete Cosine Transform

JPEG, etc:

$$DCT(i,j) = \frac{1}{\sqrt{2N}} C(i)C(j) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} Pixel(x,y) COS\left[\frac{(2x+1)i\pi}{2N}\right] COS\left[\frac{(2y+1)j\pi}{2N}\right]$$

Where $C(x) = \frac{1}{\sqrt{2}}$ if x is 0, else 1 if $x > 0$.

8 2D Inverse Discrete Cosine Transform

$$Pixel(x,y) = \frac{1}{\sqrt{2N}} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} C(i)C(j) DCT(i,j) COS\left[\frac{(2x+1)i\pi}{2N}\right] COS\left[\frac{(2y+1)j\pi}{2N}\right]$$

Where $C(x) = \frac{1}{\sqrt{2}}$ if x is 0, else 1 if $x > 0$.

9 Conclusion

In class: How JPEG works.